Expanding brackets and simplifying expressions

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions - basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form ax + b, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand 4(3x - 2)

4(3x - 2) = 12x - 8	Multiply everything inside the bracket by the 4 outside the bracket
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Example 2 Expand and simplify 3(x+5) - 4(2x+3)

3(x+5) - 4(2x+3) = 3x + 15 - 8x - 12	1 Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4
= 3 - 5x	2 Simplify by collecting like terms: 3x - 8x = -5x and $15 - 12 = 3$

Example 3 Expand and simplify (x + 3)(x + 2)

(x+3)(x+2) = x(x+2) + 3(x+2)	1 Expand the brackets by multiplying $(x+2)$ by x and $(x+2)$ by 3
$= x^{2} + 2x + 3x + 6$	2 Simplify by collecting like terms:
= $x^{2} + 5x + 6$	2x + 3x = 5x

Example 4 Expand and simplify (x - 5)(2x + 3)

(x-5)(2x+3) = x(2x+3) - 5(2x+3)	1 Expand the brackets by multiplying $(2x+3)$ by x and $(2x+3)$ by -5
$= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	2 Simplify by collecting like terms: 3x - 10x = -7x



Practice

1	Exp	band.			Watch out!
	a	3(2x-1)	b	$-2(5pq+4q^2)$	When multiplying (or
	c	$-(3xy-2y^2)$			dividing) positive and
2	Exp	and and simplify.			negative numbers, if
	a	7(3x+5)+6(2x-8)	b	8(5p-2) - 3(4p+9)	the signs are the same the answer is $+$ if the
	c	9(3s+1)-5(6s-10)	d	2(4x-3) - (3x+5)	signs are different the
_	-				answer is '-'.
3	Exp	band. $(1 + 0)$			
	a	3x(4x+8)	b	$4k(5k^2-12)$	
	c	$-2h(6h^2+11h-5)$	d	$-3s(4s^2 - 7s + 2)$	
4	Exp	and and simplify.			
	a	$3(y^2 - 8) - 4(y^2 - 5)$	b	2x(x+5) + 3x(x-7)	
	c	4p(2p-1) - 3p(5p-2)	d	3b(4b-3) - b(6b-9)	
5	Ex	pand $\frac{1}{2}(2y-8)$			
		_			
6	Exp	and and simplify.			
	a	13 - 2(m + 7)	b	$5p(p^2+6p)-9p(2p-3)$	
7	The	e diagram shows a rectangle.			
	Wri the	te down an expression, in terms of : rectangle.	x, for	the area of $3x-5$	
	Sho	w that the area of the rectangle can	be w	ritten as	
	21x	$x^{2}-35x$			7x
8	Exp	and and simplify.			
	a	(x+4)(x+5)	b	(x+7)(x+3)	
	c	(x+7)(x-2)	d	(x+5)(x-5)	
	e	(2x+3)(x-1)	f	(3x-2)(2x+1)	
	g	(5x-3)(2x-5)	h	(3x-2)(7+4x)	
	i	(3x+4y)(5y+6x)	j	$(x+5)^2$	
	k	$(2x-7)^2$	1	$(4x-3y)^2$	
Ex	ten	d			
9	Exp	band and simplify $(x+3)^2 + (x-4)^2$			
10	Expand and simplify.				

a
$$\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$$
 b $\left(x+\frac{1}{x}\right)^2$

Pearson

Answers

1	a	6x - 3	b	$-10pq - 8q^2$
	c	$-3xy + 2y^2$		
2	a	21x + 35 + 12x - 48 = 33x - 13		
	b	40p - 16 - 12p - 27 = 28p - 43		
	c	27s + 9 - 30s + 50 = -3s + 59 = 5	9 – 3	S
	d	8x - 6 - 3x - 5 = 5x - 11		
3	a	$12x^2 + 24x$	b	$20k^3 - 48k$
	c	$10h - 12h^3 - 22h^2$	d	$21s^2 - 21s^3 - 6s$
4	a	$-y^2 - 4$	b	$5x^2 - 11x$
	c	$2p - 7p^2$	d	$6b^{2}$
5	<i>v</i> –	4		
	2			
6	a	-1 - 2m	b	$5p^3 + 12p^2 + 27p$
7	7x($(3x-5) = 21x^2 - 35x$		
-	(
8	a	$x^2 + 9x + 20$	b	$x^2 + 10x + 21$
	c	$x^2 + 5x - 14$	d	$x^2 - 25$
	e	$2x^2 + x - 3$	f	$6x^2 - x - 2$
	g	$10x^2 - 31x + 15$	h	$12x^2 + 13x - 14$
	i	$18x^2 + 39xy + 20y^2$	i	$x^2 + 10x + 25$
	k	$4x^2 - 28x + 49$	J	$16x^2 - 24xy + 9y^2$
			•	1000 = 200y + yy

9
$$2x^2 - 2x + 25$$

10 a
$$x^2 - 1 - \frac{2}{x^2}$$
 b $x^2 + 2 + \frac{1}{x^2}$

Surds and rationalising the denominator

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions - basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd \sqrt{b}
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	1 Choose two numbers that are factors of 50. One of the factors must be a square number
$=\sqrt{25} \times \sqrt{2}$	2 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$=5 \times \sqrt{2}$	3 Use $\sqrt{25} = 5$
$=5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	1 Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$=\sqrt{49}\times\sqrt{3}-2\sqrt{4}\times\sqrt{3}$	2 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$=7\times\sqrt{3}-2\times2\times\sqrt{3}$	3 Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	4 Collect like terms





Example 3 Simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$ $= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$ = 7 - 2 = 51 Expand the brackets. A common mistake here is to write $(\sqrt{7})^2 = 49$ 2 Collect like terms: $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise
$$\frac{1}{\sqrt{3}}$$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	1 Multiply the numerator and denominator by $\sqrt{3}$
$=\frac{1\times\sqrt{3}}{\sqrt{9}}$	2 Use $\sqrt{9} = 3$
$=\frac{\sqrt{3}}{3}$	

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$$

$$= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$$

$$= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$$

$$= \frac{\sqrt{2} \sqrt{2} \sqrt{3}}{12}$$

$$= \frac{\sqrt{2} \sqrt{2}}{12}$$

$$= \frac{\sqrt{2} \sqrt{3}}{6}$$
1 Multiply the numerator and denominator by $\sqrt{12}$
2 Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
3 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
4 Use $\sqrt{4} = 2$
5 Simplify the fraction:

$$\frac{2}{12}$$
 simplifies to $\frac{1}{6}$



Rationalise and simplify $\frac{3}{2+\sqrt{5}}$			
$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	1	Multiply the numerator and denominator by $2 - \sqrt{5}$	
$=\frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	2	Expand the brackets	
$=\frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	3	Simplify the fraction	
$= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5}-6$	4	Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1	
	Rationalise and simplify $\frac{3}{2+\sqrt{5}}$ $\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ $= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$ $= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ $= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5}-6$	Rationalise and simplify $\frac{3}{2+\sqrt{5}}$ $\begin{vmatrix} 3\\2+\sqrt{5} \\ = \\ \frac{3}{2+\sqrt{5}} \\ = \\ \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} \\ = \\ \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5} \\ = \\ \frac{6-3\sqrt{5}}{-1} \\ = \\ 3\sqrt{5}-6 \end{vmatrix}$	

Practice

1	Sim	plify.			Hint
	a	$\sqrt{45}$	b	$\sqrt{125}$	One of the two
	c	$\sqrt{48}$	d	$\sqrt{175}$	numbers you
	e	$\sqrt{300}$	f	$\sqrt{28}$	must be a square
	g	$\sqrt{72}$	h	$\sqrt{162}$	number.

2	Sin	nplify.
	a	$\sqrt{72} + \sqrt{162}$

- c $\sqrt{50} \sqrt{8}$
- e $2\sqrt{28} + \sqrt{28}$

b	$\sqrt{45} - 2\sqrt{5}$
d	$\sqrt{75} - \sqrt{48}$
f	$2\sqrt{12} - \sqrt{12} + \sqrt{27}$

Watch out!

Check you have chosen the highest square number at the start.

- 3 Expand and simplify. a $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$
 - c $(4-\sqrt{5})(\sqrt{45}+2)$

b $(3+\sqrt{3})(5-\sqrt{12})$ **d** $(5+\sqrt{2})(6-\sqrt{8})$



4 Rationalise and simplify, if possible.

a
$$\frac{1}{\sqrt{5}}$$
b $\frac{1}{\sqrt{11}}$ c $\frac{2}{\sqrt{7}}$ d $\frac{2}{\sqrt{8}}$ e $\frac{2}{\sqrt{2}}$ f $\frac{5}{\sqrt{5}}$ g $\frac{\sqrt{8}}{\sqrt{24}}$ h $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a
$$\frac{1}{3-\sqrt{5}}$$
 b $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$

Extend

- 6 Expand and simplify $(\sqrt{x} + \sqrt{y})(\sqrt{x} \sqrt{y})$
- 7 Rationalise and simplify, if possible.

a
$$\frac{1}{\sqrt{9}-\sqrt{8}}$$
 b $\frac{1}{\sqrt{x}-\sqrt{y}}$



Answers

1	a	3√5	b	5√5		
	c	$4\sqrt{3}$	d	5√7		
	e	10√3	f	2√7		
	g	6√2	h	9√2		
2				F		
Z	a	1572	D	√5 /5		
	C	3√2	d	√3 		
	e	6√7	f	5√3		
3	я	-1	b	$9 - \sqrt{3}$		
-	c	$10\sqrt{5} - 7$	ď	26-4-12		
	L	1043 /	u	20-192		
1	9	$\sqrt{5}$	h	$\sqrt{11}$		
-	a	5	U	11		
	c	$\frac{2\sqrt{7}}{7}$	d	$\frac{\sqrt{2}}{2}$		
	e	$\sqrt{2}$	f	∠ √5		
	-	√- √3		1		
	g	$\frac{\sqrt{3}}{3}$	h	$\frac{1}{3}$		
5	a	$3+\sqrt{5}$	b	$2(4-\sqrt{3})$	с	$6(5+\sqrt{2})$
		4		13		23
6	x -	у				





Rules of indices

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions - basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$ $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*

•
$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

•
$$a^{-m} = \frac{1}{a^m}$$

The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$. •

Examples

Example 1 Evaluate 10⁰

$10^0 = 1$	Any value raised to the power of zero is equal to 1
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Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Evaluate $27^{\frac{2}{3}}$ Example 3

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2$	1 Use the rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$
$= 3^{2}$ = 9	2 Use $\sqrt[3]{27} = 3$



Example 4	Evaluate 4^{-2}	
	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$	1 Use the rule $a^{-m} = \frac{1}{a^m}$ 2 Use $4^2 = 16$
Example 5	Simplify $\frac{6x^5}{2x^2}$	
	$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to
		give $\frac{x^{2}}{x^{2}} = x^{5-2} = x^{3}$
Example 6	Simplify $\frac{x^3 \times x^5}{x^4}$	
	$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$	1 Use the rule $a^m \times a^n = a^{m+n}$
	$=x^{8-4}=x^{4}$	2 Use the rule $\frac{a^m}{a^n} = a^{m-n}$
Example 7	Write $\frac{1}{3x}$ as a single power of x	
	$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the
		fraction $\frac{1}{3}$ remains unchanged
Example 8	Write $\frac{4}{\sqrt{x}}$ as a single power of x	
	$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$	1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
	$=4x^{-\frac{1}{2}}$	2 Use the rule $\frac{1}{a^m} = a^{-m}$



Practice

1	Evaluate.						
	a 14°	b	3 ⁰	c	5^{0}	d	x^0
2	Evaluate.						
	$\frac{1}{10^2}$	h	$64^{\frac{1}{3}}$	0	$125^{\frac{1}{3}}$	d	$16\frac{1}{4}$
	a 45 ⁻	U	04*	Ľ	123	u	10
2	Frielingto						
3	Evaluate.		5		3		3
	a $25^{\overline{\overline{2}}}$	b	$8^{\frac{5}{3}}$	c	$49^{\frac{3}{2}}$	d	$16^{\frac{3}{4}}$
4	Evaluate.						
	a 5 ⁻²	b	4 ⁻³	c	2^{-5}	d	6 ⁻²
5	Simplify.						
	$\frac{3x^2 \times x^3}{3x^2 \times x^3}$	h	$10x^{5}$				
	$2x^2$	D	$2x^2 \times x$				
	$\frac{3x \times 2x^3}{2x^3}$	d	$7x^3y^2$		Watah out!		
	$2x^3$	u	$14x^5y$		watch out:		
	y^2	£	$c^{rac{1}{2}}$		Remember the	at ad to	
	e $\overline{y^{\frac{1}{2}} \times y}$	I	$\overline{c^2 \times c^{\frac{3}{2}}}$		the power of z	zero	
	$(2u^2)^3$		1 3		is 1. This is th	e	
	$\mathbf{g} = \frac{\left(\frac{2x}{4}\right)}{4}$	h	$\frac{x^2 \times x^2}{-2}$		rule $a^0 = 1$.		
	$4x^{\circ}$		$x^2 \times x^3$				
-							
6	Evaluate.		2				
	a $4^{-\frac{1}{2}}$	b	$27^{-\frac{2}{3}}$	c	$9^{-\frac{1}{2}} \times 2^{3}$		
	1		$(a)^{-\frac{1}{2}}$		$(2\pi)^{-\frac{2}{2}}$		
	$\mathbf{d} \qquad 16^{\overline{4}} \times 2^{-3}$	e	$\left(\frac{9}{16}\right)^2$	f	$\left(\frac{27}{64}\right)^{-3}$		
			(10)		(04)		
_							
7	Write the following as a	single	power of x .				
	$\mathbf{a} = \frac{1}{2}$	b	$\frac{1}{7}$	c	$\sqrt[4]{x}$		
	x		x^{\cdot}				

e $\frac{1}{\sqrt[3]{x}}$

 $\mathbf{f} \qquad \frac{1}{\sqrt[3]{x^2}}$

d $\sqrt[5]{x^2}$



8 Write the following without negative or fractional powers.

a	x^{-3}	b x^0	c	$x^{\frac{1}{5}}$
d	$x^{\frac{2}{5}}$	e $x^{-\frac{1}{2}}$	f	$x^{-\frac{3}{4}}$

Wr	tite the following in th	ne form	ax^n .		
a	$5\sqrt{x}$	b	$\frac{2}{x^3}$	c	$\frac{1}{3x^4}$
d	$\frac{2}{\sqrt{x}}$	e	$\frac{4}{\sqrt[3]{x}}$	f	3

Extend

9

10 Write as sums of powers of *x*.

a
$$\frac{x^5+1}{x^2}$$
 b $x^2\left(x+\frac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\frac{1}{x^3}\right)$



Answers

1	a	1	b	1	c	1	d	1
2	a	7	b	4	c	5	d	2
3	a	125	b	32	c	343	d	8
4	a	$\frac{1}{25}$	b	$\frac{1}{64}$	c	$\frac{1}{32}$	d	$\frac{1}{36}$
5	a	$\frac{3x^3}{2}$	b	$5x^2$				
	c	3 <i>x</i>	d	$\frac{y}{2x^2}$				
	e g	$y^{\frac{1}{2}}$ $2x^{6}$	f h	c ⁻³ x				
6	a	$\frac{1}{2}$	b	$\frac{1}{9}$	c	$\frac{8}{3}$		
	d	$\frac{1}{4}$	e	$\frac{4}{3}$	f	$\frac{16}{9}$		
7	a	<i>x</i> ⁻¹	b	x ⁻⁷ .	c	$x^{\frac{1}{4}}$		
	d	$x^{\frac{2}{5}}$	e	$x^{-\frac{1}{3}}$	f	$x^{-\frac{2}{3}}$		
8	a	$\frac{1}{x^3}$	b	1	c	$\sqrt[5]{x}$		
	d	$\sqrt[5]{x^2}$	e	$\frac{1}{\sqrt{x}}$	f	$\frac{1}{\sqrt[4]{x^3}}$		
9	a	$5x^{\frac{1}{2}}$	b	$2x^{-3}$	c	$\frac{1}{3}x^{-4}$		
	d	$2x^{-\frac{1}{2}}$	e	$4x^{-\frac{1}{3}}$	f	$3x^{0}$		
10	a	$x^3 + x^{-2}$	b	$x^3 + x$	c	$x^{-2} + x^{-7}$		



Factorising expressions

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions - factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 y^2$ is called the difference of two squares. It factorises to (x y)(x + y).

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$
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Example 3 Factorise $x^2 + 3x - 10$

b = 3, ac = -10	1 Work out the two factors of ac = -10 which add to give $b = 3(5 and -2)$
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	 2 Rewrite the b term (3x) using these two factors
=x(x+5)-2(x+5)	3 Factorise the first two terms and the last two terms
= (x+5)(x-2)	4 $(x+5)$ is a factor of both terms



Example 4 Factorise $6x^2 - 11x - 10$

b = -11, ac = -60	1 Work out the two factors of
So	ac = -60 which add to give $b = -11(-15 and 4)$
$6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$	2 Rewrite the <i>b</i> term $(-11x)$ using
	these two factors
= 3x(2x-5) + 2(2x-5)	3 Factorise the first two terms and the
	last two terms
=(2x-5)(3x+2)	4 $(2x-5)$ is a factor of both terms

Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	1 Factorise the numerator and the denominator
For the numerator: b = -4, $ac = -21$	2 Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
$x^{2} - 4x - 21 = x^{2} - 7x + 3x - 21$	3 Rewrite the <i>b</i> term $(-4x)$ using these two factors
=x(x-7)+3(x-7)	4 Factorise the first two terms and the last two terms
= (x-7)(x+3)	5 $(x-7)$ is a factor of both terms
For the denominator: b = 9, $ac = 18$	6 Work out the two factors of ac = 18 which add to give $b = 9(6 and 3)$
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	7 Rewrite the <i>b</i> term (9 <i>x</i>) using these two factors
=2x(x+3)+3(x+3)	8 Factorise the first two terms and the last two terms
= (x+3)(2x+3)	9 $(x+3)$ is a factor of both terms
$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$	10 $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1



Practice

1	Fac	ctorise.		
	a	$6x^4y^3 - 10x^3y^4$	b	$21a^3b^5 + 35a^5b^2$
	c	$25x^2y^2 - 10x^3y^2 + 15x^2y^3$		
2	Fac	ctorise		
	a	$x^2 + 7x + 12$	b	$x^2 + 5x - 14$
	c	$x^2 - 11x + 30$	d	$x^2 - 5x - 24$
	e	$x^2 - 7x - 18$	f	$x^2 + x - 20$
	g	$x^2 - 3x - 40$	h	$x^2 + 3x - 28$
3	Fac	ctorise		
	a	$36x^2 - 49y^2$	b	$4x^2 - 81y^2$

- **a** $36x^2 49y^2$ **b c** $18a^2 - 200b^2c^2$
- 4 Factorise

a	$2x^2 + x - 3$	b	$6x^2 + 17x + 5$
c	$2x^2 + 7x + 3$	d	$9x^2 - 15x + 4$
e	$10x^2 + 21x + 9$	f	$12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a	$\frac{2x^2 + 4x}{x^2 - x}$	b	$\frac{x^2+3x}{x^2+2x-3}$
c	$\frac{x^2-2x-8}{x^2-4x}$	d	$\frac{x^2 - 5x}{x^2 - 25}$
e	$\frac{x^2-x-12}{x^2-4x}$	f	$\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a
$$\frac{9x^2 - 16}{3x^2 + 17x - 28}$$

b $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$
c $\frac{4 - 25x^2}{10x^2 - 11x - 6}$
d $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify
$$\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$$

Hint

Take the highest common factor outside the bracket.



Answers

1	a	$2x^3y^3(3x-5y)$	b	$7a^3b^2(3b^3+5a^2)$
	c	$5x^2y^2(5-2x+3y)$		
2	a	(x+3)(x+4)	b	(x + 7)(x - 2)
	c	(x-5)(x-6)	d	(x-8)(x+3)
	e	(x-9)(x+2)	f	(x+5)(x-4)
	g	(x-8)(x+5)	h	(x + 7)(x - 4)
3	a	(6x-7y)(6x+7y)	b	(2x-9y)(2x+9y)
	c	2(3a - 10bc)(3a + 10bc)		
4	a	(x-1)(2x+3)	b	(3x+1)(2x+5)
	c	(2x+1)(x+3)	d	(3x-1)(3x-4)
	e	(5x+3)(2x+3)	f	2(3x-2)(2x-5)
5	a	2(x+2)	b	<u></u>
		x-1		x-1
	c	$\frac{x+2}{2}$	d	<u> </u>
		x		<i>x</i> + 5
	e	x+3	f	<u>x</u>
		x		x-5
		2		2 2
6	a	$\frac{3x+4}{7}$	b	$\frac{2x+3}{2}$
		x + 1		3x-2
	c	$\frac{2-5x}{2-2}$	d	$\frac{3x+1}{x+1}$
		2x-3		<i>x</i> + 4

7 (x+5)

$$8 \qquad \frac{4(x+2)}{x-2}$$



Completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions - factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x+q)^2 + r$
- If $a \neq 1$, then factorise using *a* as a common factor.

Examples

$x^2 + 6x - 2$	1 Write $x^2 + bx + c$ in the form
$=(x+3)^2-9-2$	$\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$
$=(x+3)^2-11$	2 Simplify

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

Example 2	Write $2x^2$ –	5x + 1	in the	form	p(x +	$(q)^{2} + r$
-----------	----------------	--------	--------	------	-------	---------------

$2x^2 - 5x + 1$	1 Before completing the square write $ax^2 + bx + c$ in the form
$= 2\left(x^2 - \frac{5}{2}x\right) + 1$	$a\left(x^{2} + \frac{b}{a}x\right) + c$ 2 Now complete the square by writing $x^{2} - \frac{5}{a}x$ in the form
$= 2\left[\left(x-\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$	$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$	3 Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the
$= 2\left(x-\frac{5}{4}\right)^2 - \frac{17}{8}$	factor of 2 4 Simplify



Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a	$x^2 + 4x + 3$	b	$x^2 - 10x - 3$
c	$x^2 - 8x$	d	$x^2 + 6x$
e	$x^2 - 2x + 7$	f	$x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x+q)^2 + r$ a $2x^2 - 8x - 16$ b $4x^2 - 8x - 16$

- **c** $3x^2 + 12x 9$ **d** $2x^2 + 6x 8$
- **3** Complete the square.

a	$2x^2 + 3x + 6$	b	$3x^2 - 2x$
c	$5x^2 + 3x$	d	$3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.



Answers

1	a	$(x+2)^2 - 1$	b	$(x-5)^2-28$
	c	$(x-4)^2 - 16$	d	$(x+3)^2 - 9$
	e	$(x-1)^2 + 6$	f	$\left(x+\frac{3}{2}\right)^2 - \frac{17}{4}$
2	a	$2(x-2)^2 - 24$	b	$4(x-1)^2 - 20$
	c	$3(x+2)^2 - 21$	d	$2\left(x+\frac{3}{2}\right)^2 - \frac{25}{2}$
3	a	$2\left(x+\frac{3}{4}\right)^2+\frac{39}{8}$	b	$3\left(x-\frac{1}{3}\right)^2-\frac{1}{3}$
	c	$5\left(x+\frac{3}{10}\right)^2-\frac{9}{20}$	d	$3\left(x+\frac{5}{6}\right)^2+\frac{11}{12}$

4
$$(5x+3)^2+3$$



Solving quadratic equations by factorisation

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions - factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is *b* and whose products is *ac*.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$	1 Rearrange the equation so that all of the terms are on one side of the
$5x^2 - 15x = 0$	equation and it is equal to zero.
	would lose the solution $x = 0$.
5x(x-3)=0	2 Factorise the quadratic equation.
	5x is a common factor.
So $5x = 0$ or $(x - 3) = 0$	3 When two values multiply to make
	zero, at least one of the values must
	be zero.
Therefore $x = 0$ or $x = 3$	4 Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$	1 Factorise the quadratic equation.
b = 7, ac = 12	Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	2 Rewrite the b term (7x) using these two factors.
x(x+4) + 3(x+4) = 0	3 Factorise the first two terms and the last two terms.
(x+4)(x+3) = 0	4 $(x+4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	5 When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	6 Solve these two equations.



Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ (3x + 4)(3x - 4) = 0	1 Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$
So $(3x + 4) = 0$ or $(3x - 4) = 0$	2 When two values multiply to make zero, at least one of the values must
$x = -\frac{4}{3}$ or $x = \frac{4}{3}$	be zero.3 Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

b = -5, ac = -24	1 Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3)
So $2x^2 - 8x + 3x - 12 = 0$	2 Rewrite the <i>b</i> term $(-5x)$ using these two factors.
2x(x-4) + 3(x-4) = 0	3 Factorise the first two terms and the last two terms.
(x-4)(2x+3) = 0	4 $(x-4)$ is a factor of both terms.
So $(x-4) = 0$ or $(2x+3) = 0$	5 When two values multiply to make zero, at least one of the values must
3	be zero.
$x = 4 \text{ or } x = -\frac{1}{2}$	6 Solve these two equations.

Practice

1	Solve			
	a	$6x^2 + 4x = 0$	b	$28x^2 - 21x = 0$
	c	$x^2 + 7x + 10 = 0$	d	$x^2 - 5x + 6 = 0$
	e	$x^2 - 3x - 4 = 0$	f	$x^2 + 3x - 10 = 0$
	g	$x^2 - 10x + 24 = 0$	h	$x^2 - 36 = 0$
	i	$x^2 + 3x - 28 = 0$	j	$x^2 - 6x + 9 = 0$
	k	$2x^2 - 7x - 4 = 0$	1	$3x^2 - 13x - 10 = 0$

2 Solve

- **a** $x^2 3x = 10$ **c** $x^2 + 5x = 24$ **e** x(x+2) = 2x + 25**g** $x(3x+1) = x^2 + 15$
- **b** $x^2 3 = 2x$ **d** $x^2 - 42 = x$ **f** $x^2 - 30 = 3x - 2$ **h** 3x(x - 1) = 2(x + 1)
- Hint
- Get all terms onto one side of the equation.



Solving quadratic equations by completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions - factorising, solving, graphs and the discriminants

Key points

• Completing the square lets you write a quadratic equation in the form $p(x+q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	1	Write $x^2 + bx + c = 0$ in the form
$(x+3)^2 - 9 + 4 = 0$		$\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$
$(x+3)^2 - 5 = 0$	2	Simplify.
$(x+3)^2 = 5$	3	Rearrange the equation to work out
		x. First, add 5 to both sides.
$x + 3 = \pm \sqrt{5}$	4	Square root both sides.
• •		Remember that the square root of a
$r = \pm \sqrt{5}$ 2		value gives two answers.
$\lambda = \pm \sqrt{3} = 3$	5	Subtract 3 from both sides to solve
а. Б. Б.		the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	6	Write down both solutions.

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^{2} - 7x + 4 = 0$ $2\left(x^{2} - \frac{7}{2}x\right) + 4 = 0$	1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left[\left(x-\frac{7}{4}\right)^2-\left(\frac{7}{4}\right)^2\right]+4=0$	2 Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	3 Expand the square brackets.
$2\left(x-\frac{7}{4}\right) - \frac{17}{8} = 0$	4 Simplify. <i>(continued on next page)</i>



$2\left(x-\frac{7}{4}\right)^2 = \frac{17}{8}$	5 Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$	6 Divide both sides by 2.
$x - \frac{7}{4} = \pm \frac{\sqrt{17}}{4}$	7 Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm \frac{\sqrt{17}}{4} + \frac{7}{4}$	8 Add $\frac{7}{4}$ to both sides.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	9 Write down both the solutions.

Practice

3	Solve by completing the square.					
	a	$x^2 - 4x - 3 = 0$	b	$x^2 - 10x + x^2$		
	c	$x^2 + 8x - 5 = 0$	d	$x^2 - 2x - 6$		
	e	$2x^2 + 8x - 5 = 0$	f	$5x^2 + 3x - 4$		

- 4 Solve by completing the square. **a** (x-4)(x+2) = 5
 - **b** $2x^2 + 6x 7 = 0$
 - c $x^2 5x + 3 = 0$

b	$x^2 - 10x + 4 = 0$
d	$x^2 - 2x - 6 = 0$
f	$5x^2 + 3x - 4 = 0$

Hint

Get all terms onto one side of the equation.



Solving quadratic equations by using the formula

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions - factorising, solving, graphs and the discriminants

Key points

• Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{ac}$

$$x = \frac{b \pm \sqrt{b}}{2a}$$

- If $b^2 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for *a*, *b* and *c*.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$$a = 1, b = 6, c = 4$$

 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 1 Identify a, b and c and write down
the formula.
Remember that $-b \pm \sqrt{b^2 - 4ac}$ is
all over $2a$, not just part of it. $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$ 2 Substitute $a = 1, b = 6, c = 4$ into the
formula. $x = \frac{-6 \pm \sqrt{20}}{2}$ 3 Simplify. The denominator is 2, but
this is only because $a = 1$. The
denominator will not always be 2. $x = \frac{-6 \pm 2\sqrt{5}}{2}$ 4 Simplify $\sqrt{20}$.
 $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$ $x = -3 \pm \sqrt{5}$ 5 Simplify by dividing numerator and
denominator by 2.So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$ 6 Write down both the solutions.



$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	1 Identify <i>a</i> , <i>b</i> and <i>c</i> , making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over 2 <i>a</i> , not just part of it.
$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	2 Substitute $a = 3, b = -7, c = -2$ into the formula.
$x = \frac{7 \pm \sqrt{73}}{6}$ So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$	 3 Simplify. The denominator is 6 when a = 3. A common mistake is to always write a denominator of 2. 4 Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

Practice

- 5 Solve, giving your solutions in surd form. **a** $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$
- 6 Solve the equation $x^2 7x + 2 = 0$ Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where *a*, *b* and *c* are integers.
- 7 Solve $10x^2 + 3x + 3 = 5$ Give your solution in surd form.

Hint
Get all terms onto one side of the equation.

Extend

- 8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.
 - **a** 4x(x-1) = 3x-2
 - **b** $10 = (x+1)^2$
 - **c** x(3x-1) = 10



Answers

1
 a

$$x = 0$$
 or $x = -\frac{2}{3}$
 b
 $x = 0$ or $x = \frac{3}{4}$

 c
 $x = -5$ or $x = -2$
 d
 $x = 2$ or $x = 3$

 e
 $x = -1$ or $x = 4$
 f
 $x = -5$ or $x = 2$

 g
 $x = 4$ or $x = 6$
 h
 $x = -6$ or $x = 6$

 i
 $x = -7$ or $x = 4$
 j
 $x = 3$

 k
 $x = -\frac{1}{2}$ or $x = 4$
 I
 $x = -\frac{2}{3}$ or $x = 5$

 2
 a
 $x = -2$ or $x = 5$
 b
 $x = -1$ or $x = 3$

a

$$x = -2$$
 or $x = 5$
 b
 $x = -1$ or $x = 5$

 c
 $x = -8$ or $x = 3$
 d
 $x = -6$ or $x = 7$

 e
 $x = -5$ or $x = 5$
 f
 $x = -4$ or $x = 7$

 g
 $x = -3$ or $x = 2\frac{1}{2}$
 h
 $x = -\frac{1}{3}$ or $x = 2$

3 a
$$x = 2 + \sqrt{7}$$
 or $x = 2 - \sqrt{7}$
b $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$
c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$
d $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$
e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$
f $x = \frac{-3 + \sqrt{89}}{10}$ or $x = \frac{-3 - \sqrt{89}}{10}$

4 a
$$x = 1 + \sqrt{14}$$
 or $x = 1 - \sqrt{14}$
c $x = \frac{5 + \sqrt{13}}{2}$ or $x = \frac{5 - \sqrt{13}}{2}$

b
$$x = \frac{-3 + \sqrt{23}}{2}$$
 or $x = \frac{-3 - \sqrt{23}}{2}$

 $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

or $x = 5 - \sqrt{21}$

5 **a**
$$x = -1 + \frac{\sqrt{3}}{3}$$
 or $x = -1 - \frac{\sqrt{3}}{3}$ **b**

6
$$x = \frac{7 + \sqrt{41}}{2}$$
 or $x = \frac{7 - \sqrt{41}}{2}$

7
$$x = \frac{-3 + \sqrt{89}}{20}$$
 or $x = \frac{-3 - \sqrt{89}}{20}$

8 **a**
$$x = \frac{7 + \sqrt{17}}{8}$$
 or $x = \frac{7 - \sqrt{17}}{8}$
b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$
c $x = -1\frac{2}{3}$ or $x = 2$



Sketching quadratic graphs

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions - factorising, solving, graphs and the discriminants

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.



- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the *y*-axis substitute x = 0 into the function.
- To find where the curve intersects the *x*-axis substitute y = 0 into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

Example 1 Sketch the graph of $y = x^2$.



Example 2 Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$	1 Find where the graph intersects the y-axis by substituting $x = 0$.
When $y = 0, x^2 - x - 6 = 0$	2 Find where the graph intersects the r-axis by substituting $y = 0$
(x+2)(x-3)=0	3 Solve the equation by factorising.
x = -2 or x = 3	4 Solve $(x+2) = 0$ and $(x-3) = 0$.
So, the graph intersects the <i>x</i> -axis at $(-2, 0)$ and $(3, 0)$	5 $a = 1$ which is greater than zero, so the graph has the shape:
	(continued on next page)





Practice

- 1 Sketch the graph of $y = -x^2$.
- 2 Sketch each graph, labelling where the curve crosses the axes. **a** y = (x+2)(x-1) **b** y = x(x-3) **c** y = (x+1)(x+5)
- **3** Sketch each graph, labelling where the curve crosses the axes.

a	$y = x^2 - x - 6$	b	$y = x^2 - 5x + 4$	c	$y = x^2 - 4$
d	$y = x^2 + 4x$	e	$y = 9 - x^2$	f	$y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.



Answers





b

b

e





c

c

f



3













4













Line of symmetry at x = -1.



Solving linear simultaneous equations using the elimination method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations 3x + y = 5 and x + y = 1

	-
3x + y = 5 - x + y = 1 2x = 4 So $x = 2$	1 Subtract the second equation from the first equation to eliminate the <i>y</i> term.
Using $x + y = 1$ 2 + y = 1 So $y = -1$	2 To find the value of y , substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	3 Substitute the values of x and y into both equations to check your answers.

Example 2 Solve x + 2y = 13 and 5x - 2y = 5 simultaneously.

x + 2y = 13 + 5x - 2y = 5 6x = 18 So x = 3	1	Add the two equations together to eliminate the <i>y</i> term.
Using $x + 2y = 13$ 3 + 2y = 13 So $y = 5$	2	To find the value of y , substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	3	Substitute the values of <i>x</i> and <i>y</i> into both equations to check your answers.



$(2x + 3y = 2) \times 4 \rightarrow 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow 15x + 12y = 36$ 7x = 28 So $x = 4$	1 Multiply the first equation by 4 and the second equation by 3 to make the coefficient of <i>y</i> the same for both equations. Then subtract the first equation from the second equation to eliminate the <i>y</i> term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	2 To find the value of y , substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	3 Substitute the values of <i>x</i> and <i>y</i> into both equations to check your answers.

3x + 2y = 4

Example 3

ple 3 Solve 2x + 3y = 2 and 5x + 4y = 12 simultaneously.

Practice

x - 3y = 9

Solve these simultaneous equations.

1	4x + y = 8	2	3x + y = 7
	x + y = 5		3x + 2y = 5
3	4x + y = 3 $3x - y = 11$	4	3x + 4y = 7 $x - 4y = 5$
5	2x + y = 11	6	2x + 3y = 11



Solving linear simultaneous equations using the substitution method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous **Textbook:** Pure Year 1, 3.1 Linear simultaneous equations

Key points

• The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations y = 2x + 1 and 5x + 3y = 14

5x + 3(2x + 1) = 14	1	Substitute $2x + 1$ for y into the second equation.
5x + 6x + 3 = 14 11x + 3 = 14	2	Expand the brackets and simplify.
11x = 11	3	Work out the value of <i>x</i> .
50 x - 1		
Using $y = 2x + 1$ $y = 2 \times 1 + 1$	4	To find the value of y , substitute $x = 1$ into one of the original
So $y = 3$		equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	5	Substitute the values of x and y into both equations to check your answers.

Example 5 Solve 2x - y = 16 and 4x + 3y = -3 simultaneously.

y = 2x - 164x + 3(2x - 16) = -3	1 2	Rearrange the first equation. Substitute $2x - 16$ for y into the second equation.
4x + 6x - 48 = -3	3	Expand the brackets and simplify.
10x - 485 10x = 45 So $x = 4\frac{1}{2}$	4	Work out the value of <i>x</i> .
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	5	To find the value of <i>y</i> , substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	6	Substitute the values of x and y into both equations to check your answers.



Practice

Solve these simultaneous equations.

y = x - 48 y = 2x - 32x + 5y = 435x - 3y = 119 2y = 4x + 510 2x = y - 29x + 5y = 228x - 5y = -1111 3x + 4y = 812 3y = 4x - 72x - y = -132y = 3x - 413 3x = y - 114 3x + 2y + 1 = 02y - 2x = 34y = 8 - x

Extend

15 Solve the simultaneous equations 3x + 5y - 20 = 0 and $2(x + y) = \frac{3(y - x)}{4}$.



Answers

- 1 x = 1, y = 4
- 2 x = 3, y = -2
- **3** x = 2, y = -5
- 4 $x = 3, y = -\frac{1}{2}$
- **5** x = 6, y = -1
- **6** x = -2, y = 5
- 7 x = 9, y = 5
- 8 x = -2, y = -7
- 9 $x = \frac{1}{2}, y = 3\frac{1}{2}$
- **10** $x = \frac{1}{2}, y = 3$
- 11 x = -4, y = 5
- 12 x = -2, y = -5
- **13** $x = \frac{1}{4}, y = 1\frac{3}{4}$
- 14 $x = -2, y = 2\frac{1}{2}$
- **15** $x = -2\frac{1}{2}, y = 5\frac{1}{2}$


Solving linear and quadratic simultaneous equations

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations y = x + 1 and $x^2 + y^2 = 13$

$x^{2} + (x + 1)^{2} = 13$ $x^{2} + x^{2} + x + x + 1 = 13$ $2x^{2} + 2x + 1 = 13$	 Substitute x + 1 for y into the second equation. Expand the brackets and simplify.
$2x^{2} + 2x - 12 = 0$ (2x - 4)(x + 3) = 0 So x = 2 or x = -3	3 Factorise the quadratic equation.4 Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	5 To find the value of <i>y</i> , substitute both values of <i>x</i> into one of the original equations.
So the solutions are $x = 2$, $y = 3$ and $x = -3$, $y = -2$	
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES	6 Substitute both pairs of values of x and y into both equations to check your answers.
equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	



5 3.0		
$x = \frac{3-3y}{2}$	1	Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	2	Substitute $\frac{5-3y}{2}$ for x into the
		second equation. Notice how it is
2^{2} , $5y-3y^{2}$ 12		easier to substitute for <i>x</i> than for <i>y</i> .
$2y^2 + \frac{y^2}{2} = 12$	3	Expand the brackets and simplify.
$4y^2 + 5y - 3y^2 = 24$		
$y^2 + 5y - 24 = 0$		
(y+8)(y-3) = 0	4	Factorise the quadratic equation.
So $y = -8$ or $y = 3$	5	Work out the values of <i>y</i> .
Using $2x + 3y = 5$	6	To find the value of <i>x</i> , substitute
When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$		both values of <i>y</i> into one of the
When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$		original equations.
So the solutions are		
x = 14.5, v = -8 and $x = -2, v = 3$		
Check:	7	Substitute both pairs of values of x
equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES		and y into both equations to check
and $2 \times (-2) + 3 \times 3 = 5$ YES		your answers.
equation 2: $2 \times (-8)^2 + 14.5 \times (-8) = 12$ YES		
and $2 \times (3)^2 + (-2) \times 3 = 12$ YES		

Example 2 Solve 2x + 3y = 5 and $2y^2 + xy = 12$ simultaneously.

Practice

Solve these simultaneous equations.

1	$y = 2x + 1$ $x^2 + y^2 = 10$	2	$y = 6 - x$ $x^2 + y^2 = 20$
3	$y = x - 3$ $x^2 + y^2 = 5$	4	$y = 9 - 2x$ $x^2 + y^2 = 17$
5	$y = 3x - 5$ $y = x^2 - 2x + 1$	6	$y = x - 5$ $y = x^2 - 5x - 12$
7	$y = x + 5$ $x^2 + y^2 = 25$	8	$y = 2x - 1$ $x^2 + xy = 24$
9	$y = 2x$ $y^2 - xy = 8$	10	2x + y = 11 $xy = 15$

Extend

11	x - y = 1	12	y-x=2
	$x^2 + y^2 = 3$		$x^2 + xy = 3$



Answers

x = 1, y = 31 $x = -\frac{9}{5}, y = -\frac{13}{5}$ 2 x = 2, y = 4x = 4, y = 23 x = 1, y = -2x = 2, y = -14 x = 4, y = 1 $x = \frac{16}{5}, y = \frac{13}{5}$ 5 x = 3, y = 4x = 2, y = 16 x = 7, y = 2x = -1, y = -67 x = 0, y = 5x = -5, y = 08 $x = -\frac{8}{3}, y = -\frac{19}{3}$ x = 3, y = 59 x = -2, y = -4x = 2, y = 4**10** $x = \frac{5}{2}, y = 6$ x = 3, y = 511 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$ $x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$ 12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$ $x = \frac{-1 - \sqrt{7}}{2}, y = \frac{3 - \sqrt{7}}{2}$



Solving simultaneous equations graphically

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

• You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Examples









Example 2 Solve the simultaneous equations y = x - 4 and $y = x^2 - 4x + 2$ graphically.

Practice

- 1 Solve these pairs of simultaneous equations graphically.
 - **a** y = 3x 1 and y = x + 3
 - **b** y = x 5 and y = 7 5x
 - c y = 3x + 4 and y = 2 x

2 Solve these pairs of simultaneous equations graphically.

- **a** x + y = 0 and y = 2x + 6
- **b** 4x + 2y = 3 and y = 3x 1
- c 2x + y + 4 = 0 and 2y = 3x 1

Hint

Rearrange the equation to make *y* the subject.



- **3** Solve these pairs of simultaneous equations graphically.
 - **a** y = x 1 and $y = x^2 4x + 3$
 - **b** y = 1 3x and $y = x^2 3x 3$
 - c y = 3 x and $y = x^2 + 2x + 5$
- 4 Solve the simultaneous equations x + y = 1 and $x^2 + y^2 = 25$ graphically.

Extend

- 5 a Solve the simultaneous equations 2x + y = 3 and $x^2 + y = 4$
 - i graphically
 - ii algebraically to 2 decimal places.
 - **b** Which method gives the more accurate solutions? Explain your answer.



Answers

- 1 **a** x = 2, y = 5 **b** x = 2, y = -3**c** x = -0.5, y = 2.5
- **2 a** x = -2, y = 2**b** x = 0.5, y = 0.5
 - **c** x = -1, y = -2
- 3 **a** x = 1, y = 0 and x = 4, y = 3 **b** x = -2, y = 7 and x = 2, y = -5**c** x = -2, y = 5 and x = -1, y = 4
- 4 x = -3, y = 4 and x = 4, y = -3
- 5 a i x = 2.5, y = -2 and x = -0.5, y = 4ii x = 2.41, y = -1.83 and x = -0.41, y = 3.83
 - **b** Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.



Linear inequalities

A LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $-8 \le 4x < 16$

-8 < 4x < 16	Divide all three terms by 4.
$-2 \leq x < 4$	5

Example 2 Solve $4 \le 5x < 10$

$4 \le 5x < 10$	Divide all three terms by 5.
$\frac{4}{5} \le x < 2$	

Example 3 Solve 2x - 5 < 7

2x - 5 < 7 $2x < 12$	 Add 5 to both sides. Divide both sides by 2.
x < 6	

Example 4 Solve $2 - 5x \ge -8$

$2-5x \ge -8$ $-5x \ge -10$ $x \le 2$	 Subtract 2 from both sides. Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.
---	--

Example 5 Solve 4(x-2) > 3(9-x)

4(x-2) > 3(9-x) 4x-8 > 27-3x 7x-8 > 27 7x > 35 x > 5	 Expand the brackets. Add 3x to both sides. Add 8 to both sides. Divide both sides by 7.
<i>x</i> > 5	



Practice

1	So	lve these inequalities.				
	a	4x > 16	b	$5x - 7 \le 3$	c	$1 \ge 3x + 4$
	d	5 - 2x < 12	e	$\frac{x}{2} \ge 5$	f	$8 < 3 - \frac{x}{3}$
2	So	lve these inequalities.				
	a	$\frac{x}{5} < -4$	b	$10 \ge 2x + 3$	c	7 - 3x > -5
3	So	lve				
	a	$2 - 4x \ge 18$	b	$3 \le 7x + 10 < 45$	c	$6-2x \ge 4$
	d	4x + 17 < 2 - x	e	4-5x < -3x	f	$-4x \ge 24$
4	So	lve these inequalities.				
	a	3t + 1 < t + 6		b $2(3n-1)$	$) \ge n +$	5
5	So	lve.				
	a	3(2-x) > 2(4-x)	+ 4	b $5(4-x)$	> 3(5 -	(-x) + 2

Extend

6 Find the set of values of x for which 2x + 1 > 11 and 4x - 2 > 16 - 2x.



Answers

1	a	x > 4	b	$x \le 2$	c	$x \leq -1$
	d	$x > -\frac{7}{2}$	e	$x \ge 10$	f	<i>x</i> < –15
2	a	<i>x</i> < -20	b	$x \leq 3.5$	c	<i>x</i> < 4
3	a d	$x \le -4$ $x < -3$	b e	$-1 \le x < 5$ $x > 2$	c f	$x \le 1$ $x \le -6$
4	a	$t < \frac{5}{2}$	b	$n \ge \frac{7}{5}$		
5	a	<i>x</i> < –6	b	$x < \frac{3}{2}$		

6 x > 5 (which also satisfies x > 3)



Quadratic inequalities

A LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$



Example 2 Find the set of values of x which satisfy $x^2 - 5x \le 0$

$x^{2} - 5x = 0$ $x(x - 5) = 0$ $x = 0$	1 Solve the quadratic equation by factorising.
x = 0 or x = 5	2 Sketch the graph of $y = x(x-5)$
	3 Identify on the graph where $x^2 - 5x \le 0$, i.e. where $y \le 0$
$0 \le x \le 5$	4 Write down the values which satisfy the inequality $x^2 - 5x \le 0$



 $-x^{2} - 3x + 10 = 0$ (-x + 2)(x + 5) = 0 x = 2 or x = -52 Sketch the graph of y = (-x + 2)(x + 5) = 0

3 Identify on the graph where $-x^2 - 3x + 10 \ge 0$, i.e. where $y \ge 0$

3 Write down the values which satisfy the inequality $-x^2 - 3x + 10 \ge 0$

Practice

1 Find the set of values of x for which $(x + 7)(x - 4) \le 0$

0

 $-5 \leq x \leq 2$

- 2 Find the set of values of x for which $x^2 4x 12 \ge 0$
- **3** Find the set of values of x for which $2x^2 7x + 3 < 0$
- 4 Find the set of values of x for which $4x^2 + 4x 3 > 0$
- 5 Find the set of values of x for which $12 + x x^2 \ge 0$

Extend

Find the set of values which satisfy the following inequalities.

- $\mathbf{6} \qquad x^2 + x \le \mathbf{6}$
- $7 \quad x(2x-9) < -10$
- 8 $6x^2 \ge 15 + x$





2

x

Answers

- 1 $-7 \le x \le 4$
- $2 \qquad x \le -2 \text{ or } x \ge 6$
- **3** $\frac{1}{2} < x < 3$
- 4 $x < -\frac{3}{2} \text{ or } x > \frac{1}{2}$
- 5 $-3 \le x \le 4$
- $\mathbf{6} \quad -3 \leq x \leq 2$
- 7 $2 < x < 2\frac{1}{2}$ 8 $x \le -\frac{3}{2}$ or $x \ge \frac{5}{3}$



Straight line graphs

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation y = mx + c, where *m* is the gradient and *c* is the *y*-intercept (where x = 0).
- The equation of a straight line can be written in the form ax + by + c = 0, where *a*, *b* and *c* are integers.
- When given the coordinates (*x*₁, *y*₁) and (*x*₂, *y*₂) of two points on a line the gradient is calculated using the

formula
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Example 1 A straight line has gradient $-\frac{1}{2}$ and *y*-intercept 3.

Write the equation of the line in the form ax + by + c = 0.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$	1 A straight line has equation y = mx + c. Substitute the gradient and y-intercept given in the question
$\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	 Rearrange the equation. Rearrange the equation so all the terms are on one side and 0 is on the other side. Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the *y*-intercept of the line with the equation 3y - 2x + 4 = 0.

3y - 2x + 4 = 0 3y - 2x - 4	1 Make <i>y</i> the subject of the equation.
$y = \frac{2}{3}x - \frac{4}{3}$	2 Divide all the terms by three to get the equation in the form $y =$
Gradient = $m = \frac{2}{3}$	3 In the form $y = mx + c$, the gradient is <i>m</i> and the <i>y</i> -intercept is <i>c</i> .
y-intercept = $c = -\frac{4}{3}$	



 (x_2, y_2)

y = mx + c/

gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

x

m = 3 y = 3x + c	1	Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.
$13 = 3 \times 5 + c$	2 3	Substitute the coordinates $x = 5$ and $y = 13$ into the equation. Simplify and solve the equation.
13 = 15 + c c = -2 y = 3x - 2	4	Substitute $c = -2$ into the equation y = 3x + c

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2, x_2 = 8, y_1 = 4 \text{ and } y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{8 - 2} = \frac{3}{6} = \frac{1}{2}$	1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out
$y = \frac{1}{2}x + c$ $4 = \frac{1}{2} \times 2 + c$ a = 2	 the gradient of the line. Substitute the gradient into the equation of a straight line y = mx + c. Substitute the coordinates of either point into the equation. Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	4 Simplify and solve the equation. 5 Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Practice

1 Find the gradient and the *y*-intercept of the following equations.

a	y = 3x + 5	b	$y = -\frac{1}{2}x - 7$	
c	2y = 4x - 3	d	x + y = 5	Hint Rearrange the equations
e	2x - 3y - 7 = 0	f	5x + y - 4 = 0	to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form y = mx + c.

Gradient	y-intercept	Equation of the line
5	0	
-3	2	
4	-7	



- 3 Find, in the form ax + by + c = 0 where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.
 - agradient $-\frac{1}{2}$, y-intercept -7bgradient 2, y-intercept 0cgradient $\frac{2}{3}$, y-intercept 4dgradient -1.2, y-intercept -2
- 4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.
- 5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$
- 6 Write an equation for the line passing through each of the following pairs of points.

a	(4, 5), (10, 17)	b	(0, 6), (-4, 8)
c	(-1, -7), (5, 23)	d	(3, 10), (4, 7)

Extend

7 The equation of a line is 2y + 3x - 6 = 0. Write as much information as possible about this line.



Answers

1 a
$$m = 3, c = 5$$

b $m = -\frac{1}{2}, c = -7$
c $m = 2, c = -\frac{3}{2}$
d $m = -1, c = 5$
e $m = \frac{2}{3}, c = -\frac{7}{3} \text{ or } -2\frac{1}{3}$
f $m = -5, c = 4$

2

Gradient	y-intercept	Equation of the line
5	0	y = 5x
-3	2	y = -3x + 2
4	-7	y = 4x - 7

3 a
$$x + 2y + 14 = 0$$
 b $2x - y = 0$

c
$$2x - 3y + 12 = 0$$
 d $6x + 5y + 10 = 0$

- **4** y = 4x 3
- **5** $y = -\frac{2}{3}x + 7$

6 a
$$y = 2x - 3$$
 b $y = -\frac{1}{2}x + 6$

c y = 5x - 2 **d** y = -3x + 19

7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the *y*-intercept is 3. The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $\left(4, -3\right)$.



Parallel and perpendicular lines

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation y = mx + c has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to y = 2x + 4 which passes through the point (4, 9).

y = 2x + 4 $m = 2$	1 As the lines are parallel they have the same gradient.
y = 2x + c	2 Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	3 Substitute the coordinates into the equation $y = 2x + c$
9 = 8 + c $c = 1$	4 Simplify and solve the equation.
y = 2x + 1	5 Substitute $c = 1$ into the equation y = 2x + c

Example 2 Find the equation of the line perpendicular to y = 2x - 3 which passes through the point (-2, 5).

y = 2x - 3 m = 2 $-\frac{1}{m} = -\frac{1}{2}$	1 As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = -\frac{1}{2}x + c$	2 Substitute $m = -\frac{1}{2}$ into $y = mx + c$.
$5 = -\frac{1}{2} \times (-2) + c$	3 Substitute the coordinates (-2, 5) into the equation $y = -\frac{1}{2}x + c$
5 = 1 + c c = 4	4 Simplify and solve the equation.
$y = -\frac{1}{2}x + 4$	5 Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.



Example 3 A line passes through the points (0, 5) and (9, -1).Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$$x_{1} = 0, x_{2} = 9, y_{1} = 5 \text{ and } y_{2} = -1$$

$$m = \frac{y_{2} - y_{1}}{x_{2} - x_{1}} = \frac{-1 - 5}{9 - 0}$$

$$= \frac{-6}{9} = -\frac{2}{3}$$

$$-\frac{1}{m} = \frac{3}{2}$$

$$y = \frac{3}{2}x + c$$

$$y = \frac{3}{2}x + c$$
Midpoint = $\left(\frac{0 + 9}{2}, \frac{5 + (-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$

$$2 = \frac{3}{2} \times \frac{9}{2} + c$$

$$c = -\frac{19}{4}$$

$$y = \frac{3}{2}x - \frac{19}{4}$$

$$y = \frac{3}{2}x - \frac{19}{4}$$

$$y = \frac{3}{2}x + c$$

$$1$$
Substitute the coordinates into the equation $m = \frac{y_{2} - y_{1}}{x_{2} - x_{1}}$ to work out the gradient of the line.
$$2$$
As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$$3$$
Substitute the gradient into the equation $y = mx + c$.
$$4$$
Work out the coordinates of the midpoint of the line.
$$5$$
Substitute the coordinates of the midpoint into the equation.
$$6$$
Simplify and solve the equation.
$$y = \frac{3}{2}x + c$$
.

Practice

- 1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.
 - ay = 3x + 1(3, 2)by = 3 2x(1, 3)c2x + 4y + 3 = 0(6, -3)d2y 3x + 2 = 0(8, 20)
- 2 Find the equation of the line perpendicular to $y = \frac{1}{2}x 3$ which passes through the point (-5, 3). Hint If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$
- **3** Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.
 - **a** y = 2x 6 (4, 0) **b** $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13) **c** x - 4y - 4 = 0 (5, 15) **d** 5y + 2x - 5 = 0 (6, 7)



4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (-2, -9) **b** (0, 3), (-10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a	y = 2x + 3 $y = 2x - 7$	b	y = 3x $2x + y - 3 = 0$	c	y = 4x - 3 $4y + x = 2$
d	3x - y + 5 = 0 $x + 3y = 1$	e	2x + 5y - 1 = 0 $y = 2x + 7$	f	2x - y = 6 $6x - 3y + 3 = 0$

- 6 The straight line L_1 passes through the points A and B with coordinates (-4, 4) and (2, 1), respectively.
 - **a** Find the equation of L_1 in the form ax + by + c = 0

The line L_2 is parallel to the line L_1 and passes through the point *C* with coordinates (-8, 3). **b** Find the equation of L_2 in the form ax + by + c = 0

The line L_3 is perpendicular to the line L_1 and passes through the origin.

c Find an equation of L₃



Answers

1 a y = 3x - 7 b y = -2x + 5c $y = -\frac{1}{2}x$ d $y = \frac{3}{2}x + 8$ 2 y = -2x - 73 a $y = -\frac{1}{2}x + 2$ b y = 3x + 7c y = -4x + 35 d $y = \frac{5}{2}x - 8$ 4 a $y = -\frac{1}{2}x$ b y = 2x5 a Parallel b Neither c Perpendicular d Perpendicular e Neither f Parallel 6 a x + 2y - 4 = 0 b x + 2y + 2 = 0 c y = 2x



Pythagoras' theorem

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. $c^2 = a^2 + b^2$



Example 1

Calculate the length of the hypotenuse. Give your answer to 3 significant figures.











Practice

1 Work out the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.



2 Work out the length of the unknown side in each triangle. Give your answers in surd form.





3 Work out the length of the unknown side in each triangle. Give your answers in surd form.



4 A rectangle has length 84 mm and width 45 mm. Calculate the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.



Extend

- 5 A yacht is 40 km due North of a lighthouse. A rescue boat is 50 km due East of the same lighthouse. Work out the distance between the yacht and the rescue boat. Give your answer correct to 3 significant figures.
- 6 Points A and B are shown on the diagram. Work out the length of the line AB. Give your answer in surd form.



7 A cube has length 4 cm.Work out the length of the diagonal AG.Give your answer in surd form.



Hint

Draw a diagram using the information given in the question.



Answers

1	a	10.3 cm	b	7.07 cm
	c	58.6 mm	d	8.94 cm
2	a	$4\sqrt{3}$ cm	b	$2\sqrt{21}$ cm
	c	$8\sqrt{17}$ mm	d	18√5 mm
3	a	18√13 mm	b	$2\sqrt{145}$ mm
	c	$42\sqrt{2}$ mm	d	6√89 mm
4	95.	.3 mm		

- 5 64.0 km
- 6 $3\sqrt{5}$ units
- 7 $4\sqrt{3}$ cm



Circle theorems

A LEVEL LINKS

Scheme of work: 2b. Circles - equation of a circle, geometric problems on a grid

Key points

• A chord is a straight line joining two points on the circumference of a circle. So AB is a chord.



• A tangent is a straight line that touches the circumference of a circle at only one point. The angle between a tangent and the radius is 90°.



- Two tangents on a circle that meet at a point outside the circle are equal in length. So AC = BC.
- The angle in a semicircle is a right angle. So angle $ABC = 90^{\circ}$.
- When two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.
 So angle AOB = 2 × angle ACB.









- Angles subtended by the same arc at the circumference are equal. This means that angles in the same segment are equal.
 So angle ACB = angle ADB and angle CAD = angle CBD.
- A cyclic quadrilateral is a quadrilateral with all four vertices on the circumference of a circle. Opposite angles in a cyclic quadrilateral total 180°. So x + y = 180° and p + q = 180°.
- The angle between a tangent and chord is equal to the angle in the alternate segment, this is known as the alternate segment theorem. So angle BAT = angle ACB.







Examples

Example 1 Wo man

Work out the size of each angle marked with a letter. Give reasons for your answers.



Angle $a = 360^{\circ} - 92^{\circ}$ = 268° as the angles in a full turn total 360°.	1 The angles in a full turn total 360°.
Angle $b = 268^{\circ} \div 2$ = 134° as when two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.	2 Angles <i>a</i> and <i>b</i> are subtended by the same arc, so angle <i>b</i> is half of angle <i>a</i> .



Example 2 Work out the size of the angles in the triangle. Give reasons for your answers.



Angles are 90°, $2c$ and c .	1 The angle in a semicircle is a right angle.
$90^{\circ} + 2c + c = 180^{\circ}$ $90^{\circ} + 3c = 180^{\circ}$ $3c = 90^{\circ}$ $c = 30^{\circ}$ $2c = 60^{\circ}$	2 Angles in a triangle total 180°.3 Simplify and solve the equation.
The angles are 30°, 60° and 90° as the angle in a semi-circle is a right angle and the angles in a triangle total 180°.	

Example 3 Work out the size of each angle marked with a letter. Give reasons for your answers.



Angle $d = 55^{\circ}$ as angles subtended by the same arc are equal.	Angles subtended by the same arc are equal so angle 55° and angle <i>d</i> are equal.	
Angle $e = 28^{\circ}$ as angles subtended by the same arc are equal.	2 Angles subtended by the same arc are equal so angle 28° and angle e are equal.	

Example 4 Work out the size of each angle marked with a letter. Give reasons for your answers.



Angle $f = 180^{\circ} - 94^{\circ}$ = 86° as opposite angles in a cyclic quadrilateral total 180°.	1	Opposite angles in a cyclic quadrilateral total 180° so angle 94° and angle <i>f</i> total 180° .
		(continued on next page)



Angle $g = 180^{\circ} - 86^{\circ}$ = 84° as angles on a straight line total 180°.	2 Angles on a straight line total 180° so angle <i>f</i> and angle <i>g</i> total 180° .
Angle $h = \text{angle } f = 86^{\circ}$ as angles subtended by the same arc are equal.	3 Angles subtended by the same arc are equal so angle f and angle h are equal.





Angle $i = 53^{\circ}$ because of the alternate segment theorem.	1 The angle between a tangent and chord is equal to the angle in the alternate segment
Angle $j = 53^{\circ}$ because it is the alternate angle to 53°.	 As there are two parallel lines, angle 53° is equal to angle <i>j</i> because they are alternate angles.
Angle $k = 180^{\circ} - 53^{\circ} - 53^{\circ}$ = 74° as angles in a triangle total 180°.	3 The angles in a triangle total 180°, so $i + j + k = 180^{\circ}$.

Example 6XZ and YZ are two tangents to a circle with centre O.Prove that triangles XZO and YZO are congruent.



Angle $OXZ = 90^{\circ}$ and angle $OYZ = 90^{\circ}$ as the angles in a semicircle are right angles.	For two triangles to be congruent you need to show one of the following.		
	• All three corresponding sides are equal (SSS).		
OZ is a common line and is the hypotenuse in both triangles.	• Two corresponding sides and the included angle are equal (SAS).		
OX = OY as they are radii of the same circle.	• One side and two corresponding angles are equal (ASA).		
So triangles XZO and YZO are congruent, RHS.	• A right angle, hypotenuse and a shorter side are equal (RHS).		



Practice

1 Work out the size of each angle marked with a letter. Give reasons for your answers.





b

d

b





Work out the size of each angle marked with a letter. Give reasons for your answers.



2





d



Hint

The reflex angle at point O and angle g are subtended by the same arc. So the reflex angle is twice the size of angle g.



Hint

b

d

Angle 18° and angle *h* are subtended by the same arc.

3 Work out the size of each angle marked with a letter. Give reasons for your answers.





Hint

One of the angles is in a semicircle.







a

b

4 Work out the size of each angle marked with a letter. Give reasons for your answers.



Hint

с

An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.



d





Extend

5 Prove the alternate segment theorem.



Answers

- 1 a $a = 112^\circ$, angle OAP = angle OBP = 90° and angles in a quadrilateral total 360°.
 - **b** $b = 66^{\circ}$, triangle OAB is isosceles, Angle OAP = 90^{\circ} as AP is tangent to the circle.
 - c $c = 126^{\circ}$, triangle OAB is isosceles. $d = 63^{\circ}$, Angle OBP = 90° as BP is tangent to the circle.
 - **d** $e = 44^{\circ}$, the triangle is isosceles, so angles *e* and angle OBA are equal. The angle OBP = 90° as BP is tangent to the circle.
 - $f = 92^{\circ}$, the triangle is isosceles.
 - e $g = 62^{\circ}$, triangle ABP is isosceles as AP and BP are both tangents to the circle. $h = 28^{\circ}$, the angle OBP = 90°.
- 2 **a** $a = 130^{\circ}$, angles in a full turn total 360°. $b = 65^{\circ}$, the angle at the centre of a circle is twice the angle at the circumference. $c = 115^{\circ}$, opposite angles in a cyclic quadrilateral total 180°.
 - **b** $d = 36^{\circ}$, isosceles triangle. $e = 108^{\circ}$, angles in a triangle total 180°. $f = 54^{\circ}$, angle in a semicircle is 90°.
 - c $g = 127^{\circ}$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.
 - **d** $h = 36^{\circ}$, the angle at the centre of a circle is twice the angle at the circumference.
- 3 **a** $a = 25^{\circ}$, angles in the same segment are equal. $b = 45^{\circ}$, angles in the same segment are equal.
 - **b** $c = 44^{\circ}$, angles in the same segment are equal. $d = 46^{\circ}$, the angle in a semicircle is 90° and the angles in a triangle total 180°.
 - c $e = 48^\circ$, the angle at the centre of a circle is twice the angle at the circumference. $f = 48^\circ$, angles in the same segment are equal.
 - **d** $g = 100^{\circ}$, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.
 - $h = 100^{\circ}$, angles in the same segment are equal.
- 4 **a** $a = 75^{\circ}$, opposite angles in a cyclic quadrilateral total 180°. $b = 105^{\circ}$, angles on a straight line total 180°. $c = 94^{\circ}$, opposite angles in a cyclic quadrilateral total 180°.
 - **b** $d = 92^{\circ}$, opposite angles in a cyclic quadrilateral total 180°. $e = 88^{\circ}$, angles on a straight line total 180°. $f = 92^{\circ}$, angles in the same segment are equal.
 - c $h = 80^{\circ}$, alternate segment theorem.
 - **d** $g = 35^{\circ}$, alternate segment theorem and the angle in a semicircle is 90°.



5 Angle BAT = x.

Angle $OAB = 90^{\circ} - x$ because the angle between the tangent and the radius is 90° .

OA = OB because radii are equal.

Angle OAB = angle OBA because the base of isosceles triangles are equal.

Angle AOB = $180^{\circ} - (90^{\circ} - x) - (90^{\circ} - x) = 2x$ because angles in a triangle total 180° .

Angle ACB = $2x \div 2 = x$ because the angle at the centre is twice the angle at the circumference.





Trigonometry in right-angled triangles

A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Key points

- In a right-angled triangle:
 - the side opposite the right angle is called the hypotenuse
 - the side opposite the angle θ is called the opposite
 - o the side next to the angle θ is called the adjacent.



- In a right-angled triangle:
 - the ratio of the opposite side to the hypotenuse is the sine of angle θ , $\sin \theta = \frac{\text{opp}}{\text{hyp}}$
 - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ , $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
 - the ratio of the opposite side to the adjacent side is the tangent of angle θ , $\tan \theta = \frac{\text{opp}}{\text{adj}}$
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: sin⁻¹, cos⁻¹, tan⁻¹.
- The sine, cosine and tangent of some angles may be written exactly.

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	



Examples

Example 1

Calculate the length of side *x*. Give your answer correct to 3 significant figures.





Example 2Calculate the size of angle x.Give your answer correct to 3 significant figures.








Example 3 Calculate the exact size of angle *x*.





Practice

1 Calculate the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.





2 Calculate the size of angle *x* in each triangle. Give your answers correct to 1 decimal place.



3 Work out the height of the isosceles triangle. Give your answer correct to 3 significant figures.

Hint:

Split the triangle into two right-angled triangles.

4 Calculate the size of angle θ . Give your answer correct to 1 decimal place.

Hint:

First work out the length of the common side to both triangles, leaving your answer in surd form.

5 Find the exact value of x in each triangle.





x

Pearson

The cosine rule

A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs Textbook: Pure Year 1, 9.1 The cosine rule

Key points

• *a* is the side opposite angle A. *b* is the side opposite angle B. *c* is the side opposite angle C.



- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 2bc \cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \frac{b^2 + c^2 a^2}{2bc}$.

Examples

Example 4 Work out the length of side *w*. Give your answer correct to 3 significant figures.







Example 5Work out the size of angle θ .
Give your answer correct to 1 decimal place.





Practice

6 Work out the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.





7 Calculate the angles labelled θ in each triangle. Give your answer correct to 1 decimal place.



- 8 a Work out the length of WY. Give your answer correct to 3 significant figures.
 - **b** Work out the size of angle WXY. Give your answer correct to 1 decimal place.





The sine rule

A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs **Textbook:** Pure Year 1, 9.2 The sine rule

Key points

• *a* is the side opposite angle A. *b* is the side opposite angle B. *c* is the side opposite angle C.



- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Examples

Example 6Work out the length of side x.Give your answer correct to 3 significant figures.



$A \xrightarrow{10 \text{ cm}} c \xrightarrow{a x} B$	1 Always start by labelling the angles and sides.
$\frac{a}{\sin A} = \frac{b}{\sin B}$	2 Write the sine rule to find the side.
$\frac{x}{\sin 36^\circ} = \frac{10}{\sin 75^\circ}$	3 Substitute the values <i>a</i> , <i>b</i> , <i>A</i> and <i>B</i> into the formula.
$x = \frac{10 \times \sin 36^{\circ}}{\sin 75^{\circ}}$	4 Rearrange to make <i>x</i> the subject.
x = 6.09 cm	5 Round your answer to 3 significant figures and write the units in your answer.



Example 7 Work out the size of angle θ . Give your answer correct to 1 decimal place.





d

Practice

a

9 Find the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.













10 Calculate the angles labelled θ in each triangle. Give your answer correct to 1 decimal place.



- **11 a** Work out the length of QS. Give your answer correct to 3 significant figures.
 - **b** Work out the size of angle RQS. Give your answer correct to 1 decimal place.





Areas of triangles

A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs **Textbook:** Pure Year 1, 9.3 Areas of triangles

Key points

- *a* is the side opposite angle A. *b* is the side opposite angle B. *c* is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.

Examples

Example 8 Find the area of the triangle.









Practice

12 Work out the area of each triangle. Give your answers correct to 3 significant figures.



13 The area of triangle XYZ is 13.3 cm².Work out the length of XZ.

Hint:

Rearrange the formula to make a side the subject.



Extend

- 14 Find the size of each lettered angle or side. Give your answers correct to 3 significant figures.
 - a











c





15 The area of triangle ABC is 86.7 cm². Work out the length of BC. Give your answer correct to 3 significant figures.





Answers

1	a d	6.49 cm	b	6.93 cm	C f	2.80 cm		
	u	/4.3 11111	e	7.59 CIII	1	0.07 Cm		
2	a	36.9°	b	57.1°	c	47.0°	d	38.7°
3	5.71	l cm						
4	20.4	t°						
5	a	45°	b	1 cm	c	30°	d	$\sqrt{3}$ cm
6	a	6.46 cm	b	9.26 cm	c	70.8 mm	d	9.70 cm
7	a	22.2°	b	52.9°	c	122.9°	d	93.6°
8	a	13.7 cm	b	76.0°				
9	a	4.33 cm	b	15.0 cm	c	45.2 mm	d	6.39 cm
10	a	42.8°	b	52.8°	c	53.6°	d	28.2°
11	a	8.13 cm	b	32.3°				
12	a	18.1 cm ²	b	18.7 cm ²	c	693 mm ²		
13	5.10) cm						
14	a	6.29 cm	b	84.3°	c	5.73 cm	d	58.8°

15 15.3 cm



Rearranging equations

A LEVEL LINKS

Scheme of work: 6a. Definition, differentiating polynomials, second derivatives **Textbook:** Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1	Make <i>t</i> the subject of the formula $v = u + at$.
-----------	---

v = u + at $v - u = at$	1 Get the terms containing <i>t</i> on one side and everything else on the other side.
$t = \frac{v - u}{a}$	2 Divide throughout by <i>a</i> .

Example 2 Make *t* the subject of the formula $r = 2t - \pi t$.

$r=2t-\pi t$	1 All the terms containing <i>t</i> are already on one side and everything else is on the other side
$r = t(2 - \pi)$ $t = \frac{r}{2 - \pi}$	 Factorise as t is a common factor. Divide throughout by 2 - π.

Example 3 Make *t* the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

$\frac{t+r}{5} = \frac{3t}{2}$	1 Remove the fractions first by multiplying throughout by 10.
2t + 2r = 15t $2r = 13t$	2 Get the terms containing <i>t</i> on one side and everything else on the other side and simplify.
$t = \frac{2r}{13}$	3 Divide throughout by 13.



$r = \frac{3t+5}{t-1}$	1	Remove the fraction first by multiplying throughout by $t - 1$.
r(t-1) = 3t + 5	2	Expand the brackets.
rt - r = 3t + 5 $rt - 3t = 5 + r$	3	Get the terms containing <i>t</i> on one side and everything else on the other
t(r-3) = 5 + r	4	side. Factorise the LHS as <i>t</i> is a common
$t = \frac{5+r}{r-3}$	5	factor. Divide throughout by $r - 3$.

Example 4 Make *t* the subject of the formula $r = \frac{3t+5}{t-1}$.

Practice

Change the subject of each formula to the letter given in the brackets.

 $C = \pi d \quad [d]$ $P = 2l + 2w \quad [w]$ $D = \frac{S}{T} \quad [T]$ $p = \frac{q - r}{t} \quad [t]$ $u = at - \frac{1}{2}t \quad [t]$ $V = ax + 4x \quad [x]$ $\frac{y - 7x}{2} = \frac{7 - 2y}{3} \quad [y]$ $x = \frac{2a - 1}{3 - a} \quad [a]$ $x = \frac{b - c}{d} \quad [d]$ $\frac{2x + 3}{2}$

10
$$h = \frac{7g-9}{2+g}$$
 [g] **11** $e(9+x) = 2e+1$ [e] **12** $y = \frac{2x+3}{4-x}$ [x]

13 Make *r* the subject of the following formulae.

a
$$A = \pi r^2$$
 b $V = \frac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \frac{2}{3}\pi r^2 h$

14 Make *x* the subject of the following formulae.

a
$$\frac{xy}{z} = \frac{ab}{cd}$$
 b $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

15 Make sin *B* the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make *x* the subject of the following equations.

a
$$\frac{p}{q}(sx+t) = x-1$$

b $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$



Answers

 $w = \frac{P-2l}{2}$ **3** $T = \frac{S}{D}$ $d = \frac{C}{\pi}$ $t = \frac{2u}{2a-1}$ 6 $x = \frac{V}{a+4}$ $t = \frac{q-r}{p}$ $a = \frac{3x+1}{x+2}$ 9 $d = \frac{b-c}{x}$ y = 2 + 3x $e = \frac{1}{x+7}$ 12 $x = \frac{4y-3}{2+y}$ $g = \frac{2h+9}{7-h}$ 13 a $r = \sqrt{\frac{A}{\pi}}$ b $r = \sqrt[3]{\frac{3V}{4\pi}}$ **c** $r = \frac{P}{\pi + 2}$ **d** $r = \sqrt{\frac{3V}{2\pi h}}$ 14 a $x = \frac{abz}{cdv}$ b $x = \frac{3dz}{4\pi cpv^2}$ $\sin B = \frac{b \sin A}{a}$ $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ **a** $x = \frac{q + pt}{q - ps}$ **b** $x = \frac{3py + 2pqy}{3p - apq} = \frac{y(3 + 2q)}{3 - aq}$



Volume and surface area of 3D shapes

A LEVEL LINKS

Scheme of work: 6b. Gradients, tangents, normals, maxima and minima

Key points

- Volume of a prism = cross-sectional area × length.
- The surface area of a 3D shape is the total area of all its faces.



- Volume of a pyramid = $\frac{1}{3}$ × area of base × vertical height.
- Volume of a cylinder = $\pi r^2 h$
- Total surface area of a cylinder = $2\pi r^2 + 2\pi rh$
- Volume of a sphere = $\frac{4}{3}\pi r^3$
- Surface area of a sphere = $4\pi r^2$
- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Total surface area of a cone = $\pi r l + \pi r^2$

Examples

Example 1The triangular prism has volume 504 cm³.Work out its length.



h

$V = \frac{1}{2}bhl$ $504 = \frac{1}{2} \times 9 \times 4 \times l$	 Write out the formula for the volume of a triangular prism. Substitute known values into the formula.
$504 = 18 \times l$	3 Simplify
$l = 504 \div 18$ $= 28 \text{ cm}$	4 Rearrange to work out <i>l</i>.5 Remember the units.



Example 2 Calculate the volume of the 3D solid. Give your answer in terms of π .



Total volume = volume of hemisphere + Volume of cone = $\frac{1}{2}$ of $\frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$	1	The solid is made up of a hemisphere radius 5 cm and a cone with radius 5 cm and height 12 - 5 = 7 cm.
Total volume = $\frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$ + $\frac{1}{3} \times \pi \times 5^2 \times 7$	2	Substitute the measurements into the formula for the total volume.
$=\frac{425}{3}\pi\mathrm{cm}^3$	3	Remember the units.

b

d

Practice

1 Work out the volume of each solid. Leave your answers in terms of π where appropriate.











f a sphere with radius 7 cm



- **g** a sphere with diameter 9 cm
- **h** a hemisphere with radius 3 cm



- 2 A cuboid has width 9.5 cm, height 8 cm and volume 1292 cm³. Work out its length.
- 3 The triangular prism has volume 1768 cm³. Work out its height.



Extend

4 The diagram shows a solid triangular prism. All the measurements are in centimetres. The volume of the prism is V cm³. Find a formula for V in terms of x. Give your answer in simplified form.



5 The diagram shows the area of each of three faces of a cuboid.The length of each edge of the cuboid is a whole number of centimetres.Work out the volume of the cuboid.





- 6 The diagram shows a large catering size tin of beans in the shape of a cylinder.
 The tin has a radius of 8 cm and a height of 15 cm.
 A company wants to make a new size of tin.
 The new tin will have a radius of 6.7 cm.
 It will have the same volume as the large tin.
 Calculate the height of the new tin.
 Give your answer correct to one decimal place.
- 7 The diagram shows a sphere and a solid cylinder. The sphere has radius 8 cm.

The solid cylinder has a base radius of 4 cm and a height of h cm.

The total surface area of the cylinder is half the total surface area of the sphere.

Work out the ratio of the volume of the sphere to the volume of the cylinder.

Give your answer in its simplest form.

8 The diagram shows a solid metal cylinder. The cylinder has base radius 4x and height 3x. The cylinder is melted down and made into a sphere of radius *r*.

Find an expression for r in terms of x.







Answers

1	a	$V = 396 {\rm cm}^3$	b	$V = 75\ 000\ {\rm cm}^3$
	c	$V = 402.5 \text{ cm}^3$	d	$V = 200\pi\mathrm{cm}^3$
	e	$V = 1008\pi \mathrm{cm}^3$	f	$V=\frac{1372}{3}\pi \ \mathrm{cm}^3$
	g	$V = 121.5\pi \mathrm{cm}^3$	h	$V = 18\pi \mathrm{cm}^3$
	i	$V = 48\pi \mathrm{cm}^3$	j	$V = \frac{98}{3} \pi \mathrm{cm}^3$

- **2** 17 cm
- **3** 17 cm
- 4 $V = x^3 + \frac{17}{2}x^2 + 4x$
- 5 $60 \, \mathrm{cm}^3$
- 6 21.4 cm
- 7 32:9
- 8 $r = \sqrt[3]{36}x$



Area under a graph

A LEVEL LINKS

Scheme of work: 7b. Definite integrals and areas under curves

Key points

• To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.





b

y

chord

► x

Examples

Example 1 Estimate the area of the region between the curve y = (3 - x)(2 + x) and the x-axis from x = 0 to x = 3. Use three strips of width 1 unit.



x $y = (3-x)(2+x)$	06	1 6	24	3 0	1	Use a table to record the value of y on the curve for each value of x .
Trapezium 1: $a_1 = 6 - 0 = 6$, $b_1 = 6$ Trapezium 2: $a_2 = 6 - 0 = 6$, $b_2 = -1$ Trapezium 3: $a_3 = 4 - 0 = 4$, $a_3 = -1$	6 - 0 = 6 - 0 = 6 - 0 = 6 - 0 = 6 - 0 = 0 - 0 = 0 - 0 = 0 - 0	= 6 = 4 = 0			2	Work out the dimensions of each trapezium. The distances between the <i>y</i> -values on the curve and the <i>x</i> -axis give the values for <i>a</i> .



$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 1(6 + 6) = 6$ $\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 1(6 + 4) = 5$ $\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 1(4 + 0) = 2$	3 Work out the area of each trapezium. $h = 1$ since the width of each trapezium is 1 unit.
Area = $6 + 5 + 2 = 13$ units ²	4 Work out the total area. Remember to give units with your answer.

Example 2 Estimate the shaded area. Use three strips of width 2 units.



x 4 6 8 10 y 7 12 13 4	1 Use a table to record <i>y</i> on the curve for each value of <i>x</i> .
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	2 Use a table to record <i>y</i> on the straight line for each value of <i>x</i> .
Trapezium 1: $a_1 = 7 - 7 = 0$, $b_1 = 12 - 6 = 6$ Trapezium 2: $a_2 = 12 - 6 = 6$, $b_2 = 13 - 5 = 8$ Trapezium 3: $a_3 = 13 - 5 = 8$, $a_3 = 4 - 4 = 0$	3 Work out the dimensions of each trapezium. The distances between the <i>y</i> -values on the curve and the <i>y</i> -values on the straight line give the values for <i>a</i> .
$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 2(0 + 6) = 6$ $\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 2(6 + 8) = 14$ $\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 2(8 + 0) = 8$	4 Work out the area of each trapezium. $h = 2$ since the width of each trapezium is 2 units.
Area = $6 + 14 + 8 = 28$ units ²	5 Work out the total area. Remember to give units with your answer.



Practice

1 Estimate the area of the region between the curve y = (5 - x)(x + 2) and the x-axis from x = 1 to x = 5. Use four strips of width 1 unit.

Hint:

For a full answer, remember to include 'units²'.

Estimate the shaded area shown on the axes.Use six strips of width 1 unit.



- 3 Estimate the area of the region between the curve $y = x^2 8x + 18$ and the x-axis from x = 2 to x = 6. Use four strips of width 1 unit.
- 4 Estimate the shaded area. Use six strips of width $\frac{1}{2}$ unit.







- 5 Estimate the area of the region between the curve $y = -x^2 4x + 5$ and the *x*-axis from x = -5 to x = 1. Use six strips of width 1 unit.
- 6 Estimate the shaded area. Use four strips of equal width.



- 7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the *x*-axis from x = 2 to x = 5. Use six strips of equal width.
- 8 Estimate the shaded area. Use seven strips of equal width.





Extend

9 The curve $y = 8x - 5 - x^2$ and the line y = 2 are shown in the sketch. Estimate the shaded area using six strips of equal width.



10 Estimate the shaded area using five strips of equal width.





Answers

- 1 34 units²
- **2** 149 units²
- **3** 14 units²
- 4 $25\frac{1}{4}$ units²
- 5 35 units^2
- $6 \quad 42 \text{ units}^2$
- 7 $26\frac{7}{8}$ units²
- $8 56 units^2$
- 9 35 units²
- **10** $6\frac{1}{4}$ units²



The New A level Mathematics will build on topics from the Higher Mathematics GCSE

We assume competency in the topics detailed below.

NUMBER AND ALGEBRA

- Collecting like terms and factorising
- Solving equations with fractions
- Surds, simplifying, multiplying and rationalising denominators
- Solving linear simultaneous equations
- Solving quadratic equations (by factorising, formula and completing the square)
- Solving quadratic inequalities
- Functional notation, composite and inverse function
- Transformation of Functions
- Graphs of quadratics, cubic and reciprocal
- Rules of indices

STATISTICS

- Construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use
- Interpret, analyse and compare the distributions of data sets using appropriate measures of average (median, mean, mode and modal class) and spread (range, including consideration of outliers), quartiles and inter-quartile range
- Use and interpret scatter graphs of bivariate data; recognise correlation , draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends while knowing the dangers of so doing

PROBABILITY

- Relate relative expected frequencies to theoretical probability, using appropriate language and the 0-1 probability scale
- Apply the property that the probabilities of an exhaustive set of outcomes sum to one;
- Enumerate sets and combinations of sets systematically, using tables, grids
- Tree diagrams and Venn diagrams

GEOMETRY AND TRIGONOMETRY

- Equation of a line , Parallel and perpendicular lines
- Length of a line segment and mid-point
- Conversion graphs
- Circle theorems
- Pythagoras' Theorem 2D and 3D
- Trigonometry in right-angled triangles
- The sine rule and cosine rule
- The area of a triangle
- Bearings

Please complete worksheets as you feel appropriate

in preparation for the new course

These can be found on

http://mrbartonmaths.com/students/legacygcse/gcse-maths-takeaway.html

Links to worksheets, videos and answers ;

http://www.mrbartonmaths.com/resources/GCSE%20Revision/GCSE%20Maths%20Takeaway/4.%20 Grades%20B%20to%20A-star/86_solving-quadratics-by-factorising.pdf

http://www.mrbartonmaths.com/resources/GCSE%20Revision/GCSE%20Maths%20Takeaway/4.%20 Grades%20B%20to%20A-star/86_solving-quadratics-by-factorising.pdf

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http://www.mrbartonmaths.com/resources/GCSE%20Revision/GCSE%20Maths%20Takeaway/4.%20 Grades%20B%20to%20A-star/101_simultaneous-equations-with-a-quadratic.pdf

https://corbettmaths.files.wordpress.com/2013/02/completing-the-square-pdf1.pdf

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https://corbettmaths.files.wordpress.com/2013/02/quadratic-inequalities-pdf.pdf

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http://www.mrbartonmaths.com/resources/GCSE%20Revision/GCSE%20Maths%20Takeaway/4.%20 Grades%20B%20to%20A-star/104_sine-and-cosine-rules-area-of-triangles-using-c2bdab-sin-c.pdf

http://www.mrbartonmaths.com/resources/GCSE%20Revision/GCSE%20Maths%20Takeaway/4.%20 Grades%20B%20to%20A-star/105_3d-pythagoras.pdf

http://www.mrbartonmaths.com/resources/GCSE%20Revision/GCSE%20Maths%20Takeaway/4.%20 Grades%20B%20to%20A-star/91_cumulative-frequency-and-box-plots.pdf

http://www.mrbartonmaths.com/resources/GCSE%20Revision/GCSE%20Maths%20Takeaway/4.%20 Grades%20B%20to%20A-star/109_histograms.pdf

http://www.mathsgenie.co.uk/resources/venndiagrams.pdf